BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B69 07040

SUBJECT: Characteristic Velocity Requirements for Intraorbital Phasing and Inter-orbital Transfer Missions - Case 730

DATE: July 14, 1969

FROM: H. B. Bosch

ABSTRACT

Characteristic velocity requirements are described for two types of orbital maneuver: phasing maneuvers in a geosynchronous, circular orbit by means of elliptic phasing orbits, and non-coplanar transfer between a 250 nautical mile circular orbit and other circular orbits. The plots are intended as aids when analyzing or planning near-earth missions, such as for satellite repair and maintenance, or for logistics and resupply flights.

(NASA-CR-108986) CHARACTERISTIC VELOCITY REQUIREMENTS FOR INTRAORBITAL PHASING AND INTERORBITAL TRANSFER MISSIONS (Bellcomm, Inc.) 16 p

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MEMORANDUM FOR FILE

I. PHASING IN A CIRCULAR ORBIT

Suppose a space station to be in a circular earth orbit. Suppose also that there is a satellite (or any target location) in the same orbit, but an angular distance ϕ ($0<\phi°<360$) ahead of the station, as shown in Figure 1. If transfer time is not critical (such as for a scheduled resupply or maintenance mission) a shuttle vehicle leaving the space station can put itself into an elliptic phasing orbit by applying a tangential impulse. The period of the ellipse is chosen so that, after one revolution of the phasing orbit, the target is at the point of tangency of the two orbits. A tangential impulse, equal to the original impulse but in the opposite direction, completes the maneuver.

Depending on whether the phasing ellipse is exterior (Figure 1a) or interior (Figure 1b) to the circular orbit, the transfer time will be longer or shorter than the period of the circular orbit. The periods for the exterior and interior ellipses are given by:

$$P_e = (2 - \phi°/360)\ P_c \tag{1}$$

$$P_i = (1 - \phi°/360)\ P_c \tag{2}$$

respectively, where ϕ is the phase angle per revolution of the elliptical orbit and P_c is the period of the circular orbit. These relationships are plotted on Figure 2 for a geosynchronous orbit, i.e., P_c = 24 hours. Hevertheless, because of the way in which P_c appears in formulas (1) and (2), the same plots can be applied to any circular orbit, if the time scale is adjusted accordingly.

The impulsive velocities corresponding to (1) and (2) can be derived to be*

$$\Delta V_e = 2V_c\left[\sqrt{2-(2-\phi°/360)^{-2/3}} - 1\right] \quad (3)$$

and

$$\Delta V_i = 2V_c\left[1 - \sqrt{2-(1-\phi°/360)^{-2/3}}\right] \quad (4)$$

where V_c is the velocity of the circular orbit. These are plotted on Figure 3 for a geosynchronous orbit, for which V_c = 10,088 fps. Again, because of the way in which V_c enters formulas (3) and (4), the same plots can be used for phasing in any circular orbit, provided the velocity scale is adjusted as appropriate.

Once the outgoing leg of a phasing maneuver for $\phi°$ has been completed, the return trip is equivalent to a phasing of 360° - $\phi°$ because this is the angle by which the space station now "leads" the target point. This is reflected in Figure 4 where total characteristic velocities are shown for complete maneuvers: catching up to the target, and then returning to the space station. These velocities are minimum in the sense that the complete maneuvers are combinations of interior and exterior ellipses, whichever requires less velocity.

Finally, the choice for an interior phasing ellipse is limited by the resulting perigee height (see Figure 1b), in the following sense: the larger the phase angle is, the more eccentric the ellipse has to be and the lower the perigee height becomes. Figure 5 shows this relationship.

A comparison of Figures 2 and 3 shows that a tradeoff can be made between characteristic velocity and flight time. As an example, consider a satellite which leads the station by 90°. The phasing maneuver can be accomplished by a single revolution on an ellipse. This would require 2,260 fps on an interior ellipse and it would last 18 hours. Alternatively,

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*The premultiplier 2 appears in these formulas because each phasing maneuver consists of two impulses of equal magnitude.

the phasing could be accomplished with three revolutions of an ellipse which phases 30° per revolution. This would require only 612 fps but it would last 66 hours. Similar tradeoffs could also be made for the return trip.

II. ELLIPTIC TRANSFER BETWEEN CIRCULAR ORBITS

Transfer between two coplanar circular orbits is accomplished most economically via an elliptical orbit whose periapse and apoapse are tangent to the lower and higher circular orbits, respectively. This is the well-known Hohmann transfer and the maneuver requires two impulses.

If the two circular orbits are inclined to each other, and the apsidal line coincides with the line of intersection of the planes of the two orbits, then the impulse at apoapsis should be out-of-plane so as to accomplish the required plane change simultaneously with circularization. Figure 6 illustrates the case of transferring from one circular orbit to another, non-coplanar, higher circular orbit. The total characteristic velocity for the transfer maneuver is

$$\Delta V = V_p - V_1 + \sqrt{V_2^2 + V_a^2 - 2V_2V_a \cos \alpha} \qquad (5)$$

where V_1 and V_2 are the circular velocities of the lower and higher orbits, respectively; V_p and V_a are, respectively, perigee and apogee velocities of the transfer ellipse; and α is the mutual inclination of the two orbital planes.

Contours of constant ΔV, corresponding to formula (5), are shown in Figure 7 for transfer between a 250 nautical mile circular orbit and other circular orbits. As a special case, the characteristic velocities required for coplanar (Hohmann) transfer to and from a 250 nautical mile circular orbit are plotted in Figure 8. The characteristic velocities required for pure plane changes in circular orbits of given altitude are shown in Figure 9.

CAVEAT: Figure 7 shows characteristic velocities only for transfer to and from 250 nautical mile circular orbits. For transfer to and from other orbits, refer to the Appendix.

III. SOME OBSERVATIONS

Figure 4 shows that no phasing maneuver in geosynchronous orbit requires more than about 4500 fps to complete, if transfer time is not critical. Also, use of Figure 2 in conjunction with Figure 3 indicates that the total flight time (catching up and returning) is either 72 hours (for $105° < \phi < 255°$) or 48 hours (for $0° < \phi \leq 105°$ or $255° \leq \phi < 360°$). Of course, if transfer time becomes critical, such as for a rescue or emergency repair mission, the trajectory selected for at least the catching up maneuver could be anything from a segment of a highly eccentric ellipse to a hyperbolic arc.

In the case of elliptic transfer between two circular orbits, the contours plotted on Figure 7 show that the plane change maneuver is the dominant velocity requirement for most cases (provided, of course, that the plane change is accomplished at the apogee of the transfer ellipse, as shown in Figure 6). For example, for plane changes of around 20° or more, there is generally very little variation in the total velocity requirement as altitude increases. For small plane changes (a degree or two), however, changing altitude becomes the dominant factor.

HB Bosch

1013-HBB-ulg H. B. Bosch

Attachments
Appendix
Figures 1-9

APPENDIX

The fundamental formula for the total characteristic velocity (ΔV) required to accomplish noncoplanar transfer between two circular orbits (see Figure 6) is

$$\Delta V = V_p - V_1 + \sqrt{V_2^{\,2} + V_a^{\,2} - 2V_2V_a \cos \alpha} \tag{5}$$

where V_1 and V_2 are the velocities of the lower and higher orbits, respectively. Equivalent formulations can be written in terms of r_1 and r_2--the radii of the lower and higher orbits, respectively--as follows:

$$\frac{\Delta V}{V_1} = \sqrt{\frac{2r_2}{r_1+r_2}} - 1 + \sqrt{\frac{r_1}{r_2}}\sqrt{1 + \frac{2r_1}{r_1+r_2} - 2\sqrt{\frac{2r_1}{r_1+r_2}}\cos \alpha} \tag{A.1}$$

or

$$\frac{\Delta V}{V_2} = \sqrt{\frac{r_2}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right) + \sqrt{1 + \frac{2r_1}{r_1+r_2} - 2\sqrt{\frac{2r_1}{r_1+r_2}}\cos \alpha} \tag{A.2}$$

depending on whether V_1 or V_2 is taken as the reference velocity.

For a pure plane change ($r_1 = r_2$) both formulas reduce to

$$\frac{\Delta V}{V_1} = 2 \sin \frac{\alpha}{2} \tag{A.3}$$

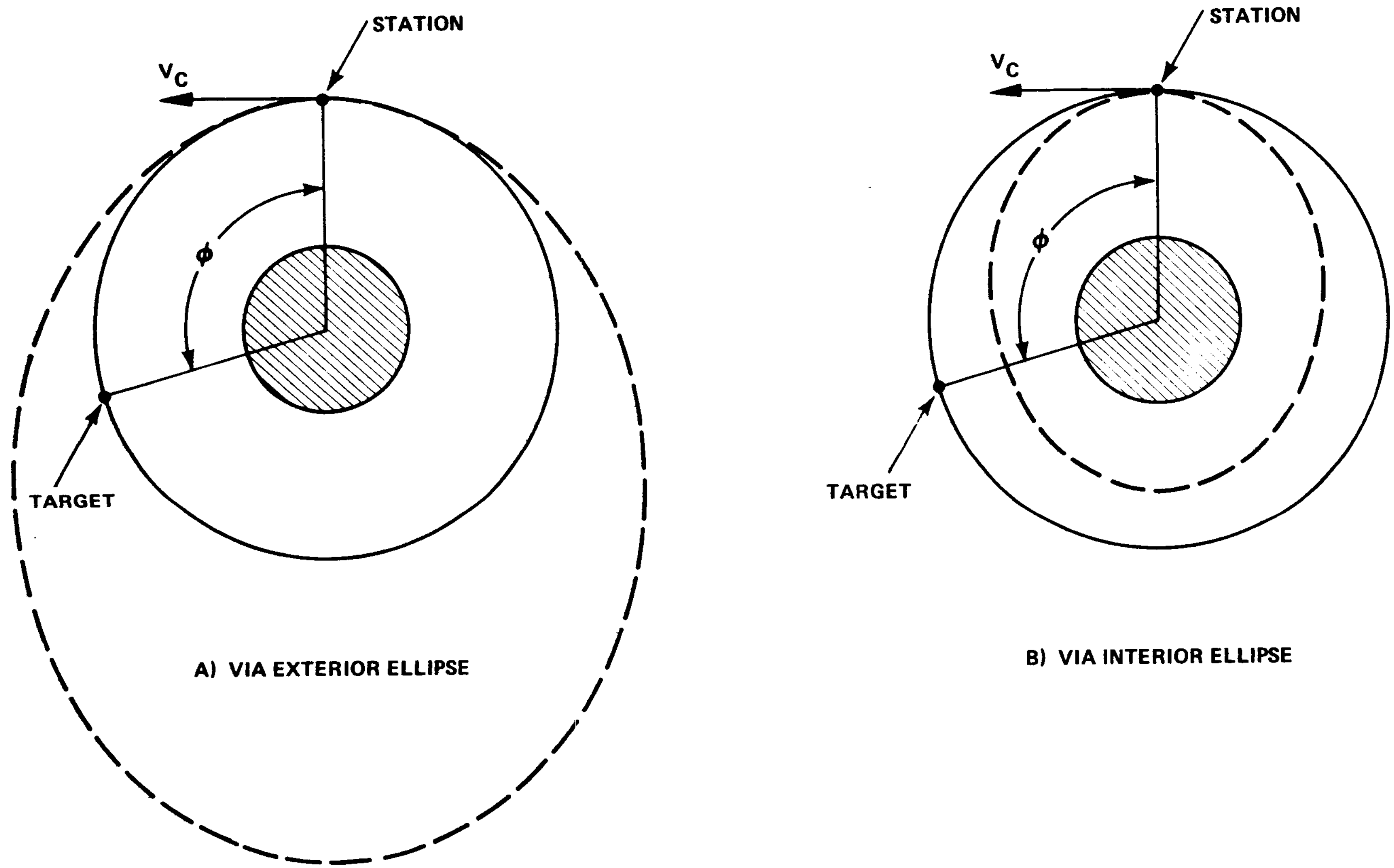

FIGURE 1 - TWO MODES OF PHASING IN CIRCULAR ORBIT

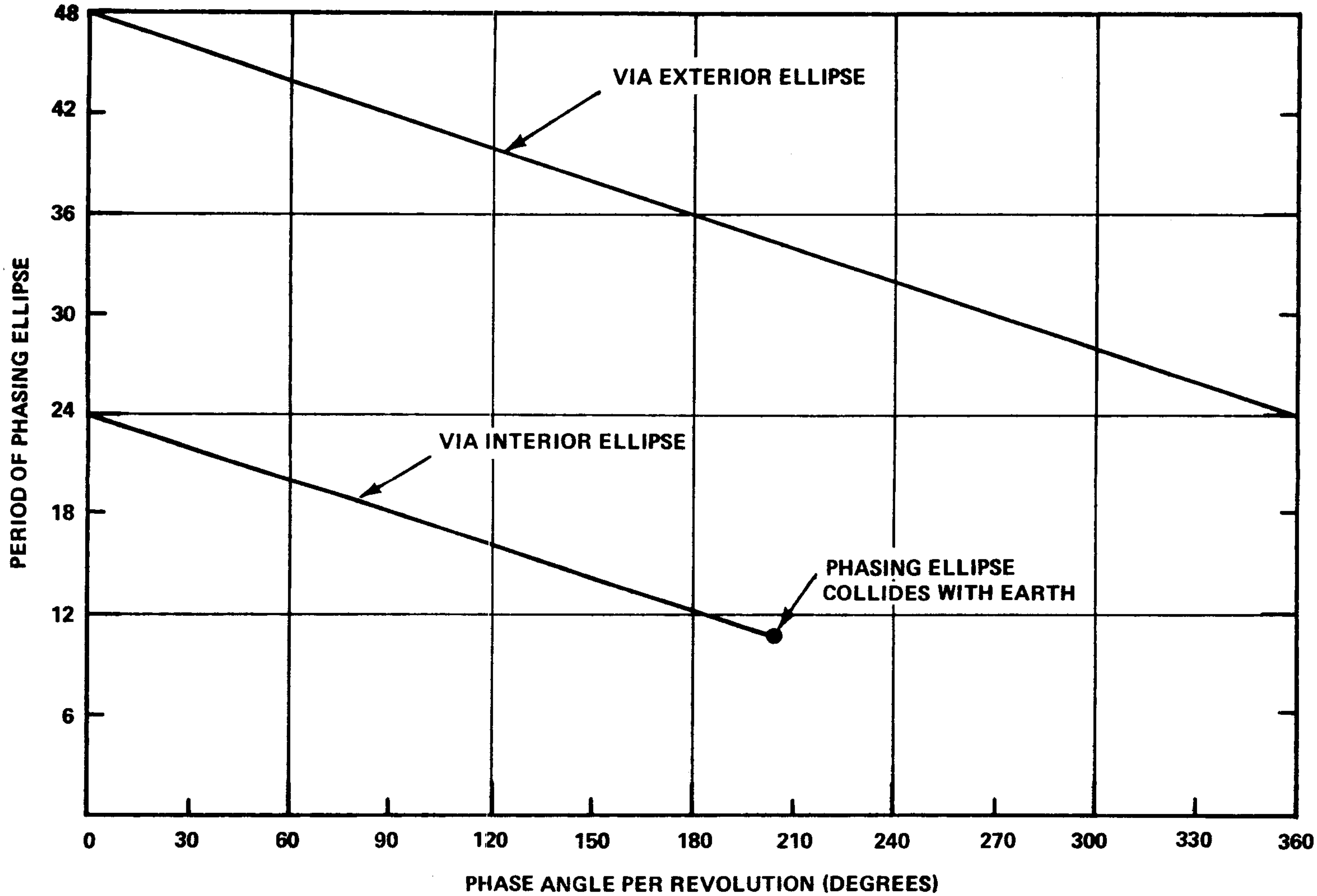

FIGURE 2- TRANSFER TIME FOR TWO-IMPULSE PHASING MANEUVERS IN GEOSYNCHRONOUS CIRCULAR ORBIT

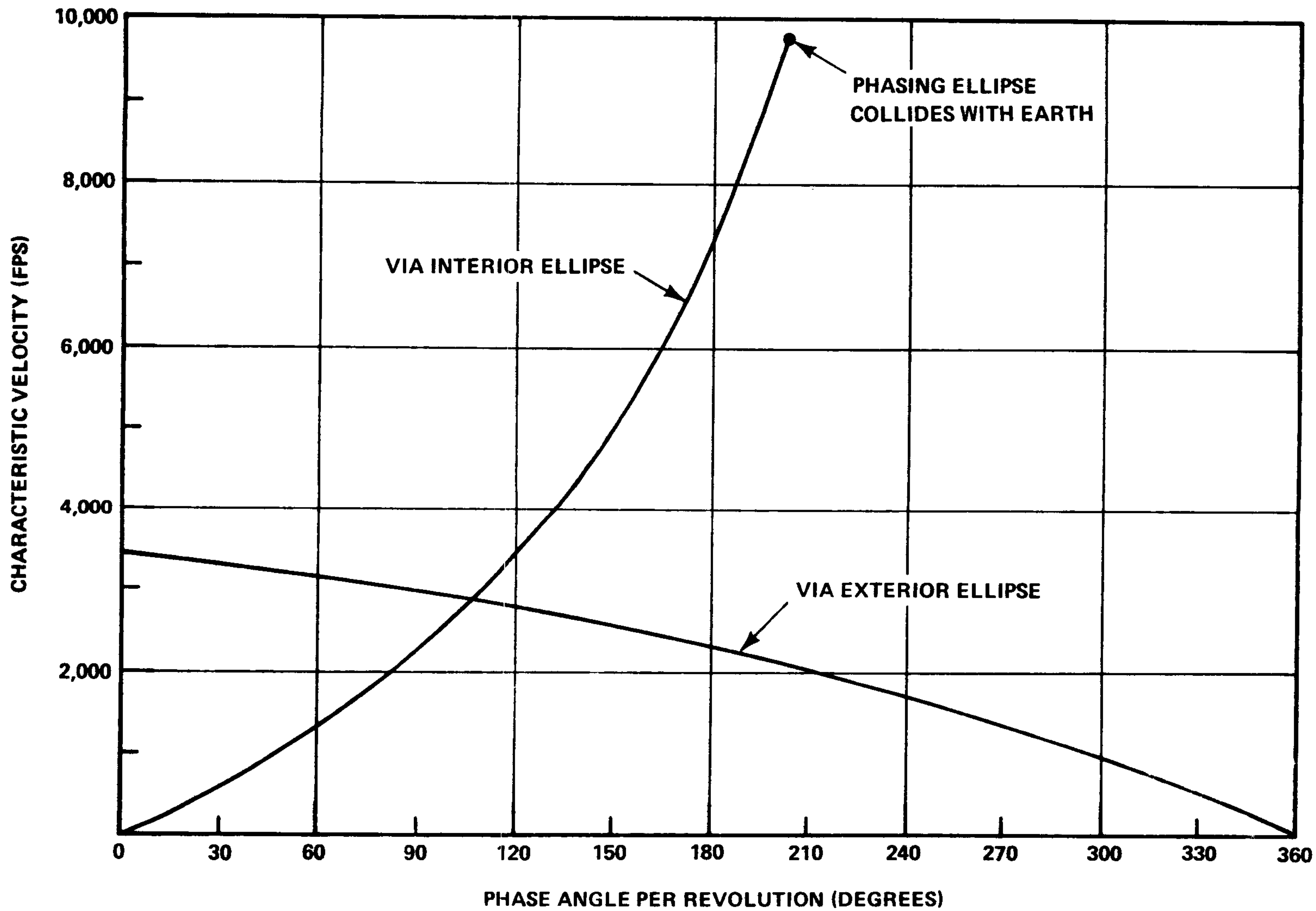

FIGURE 3- TWO-IMPULSE PHASING MANEUVERS IN GEOSYNCHRONOUS CIRCULAR ORBIT BY USE OF EXTERIOR OR INTERIOR ELLIPTIC ORBITS

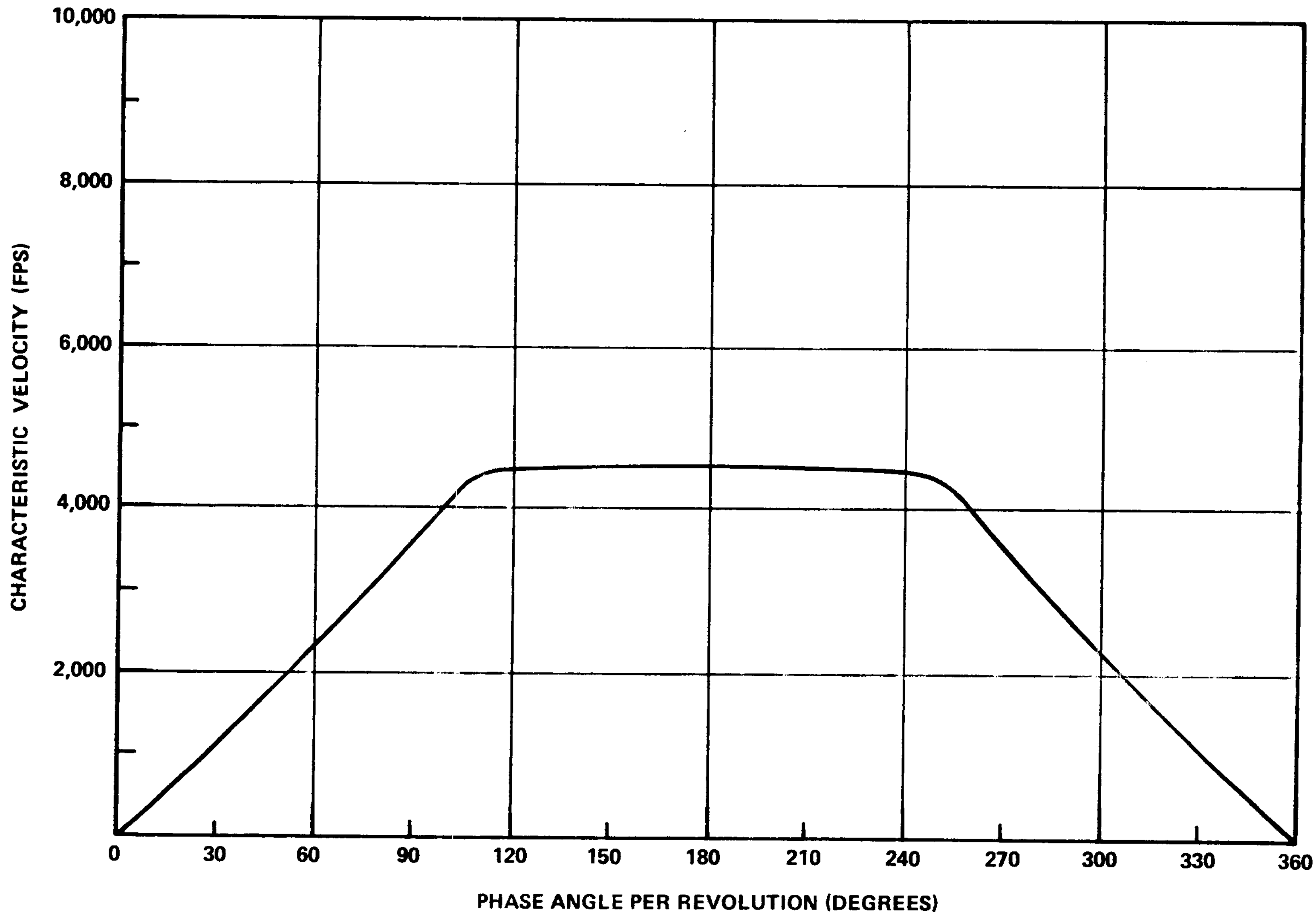

FIGURE 4- MINIMUM TOTAL VELOCITY REQUIRED FOR PHASING MANEUVERS IN GEOSYNCHRONOUS ORBIT, INCLUDING RETURN MANEUVER

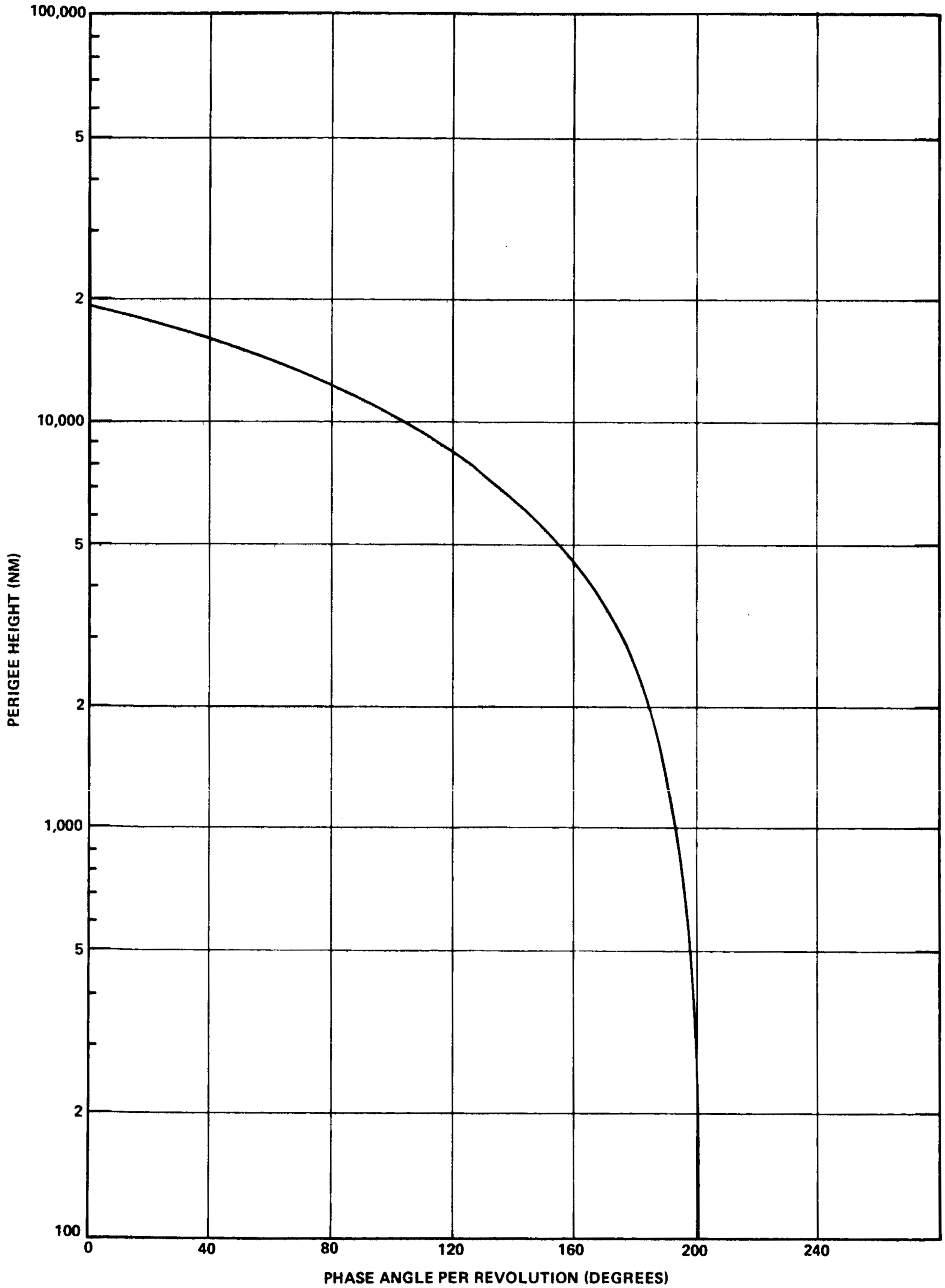

FIGURE 5 - PERIGEE HEIGHT OF INTERIOR PHASING ELLIPSE FROM GEOSYNCHRONOUS CIRCULAR ORBIT

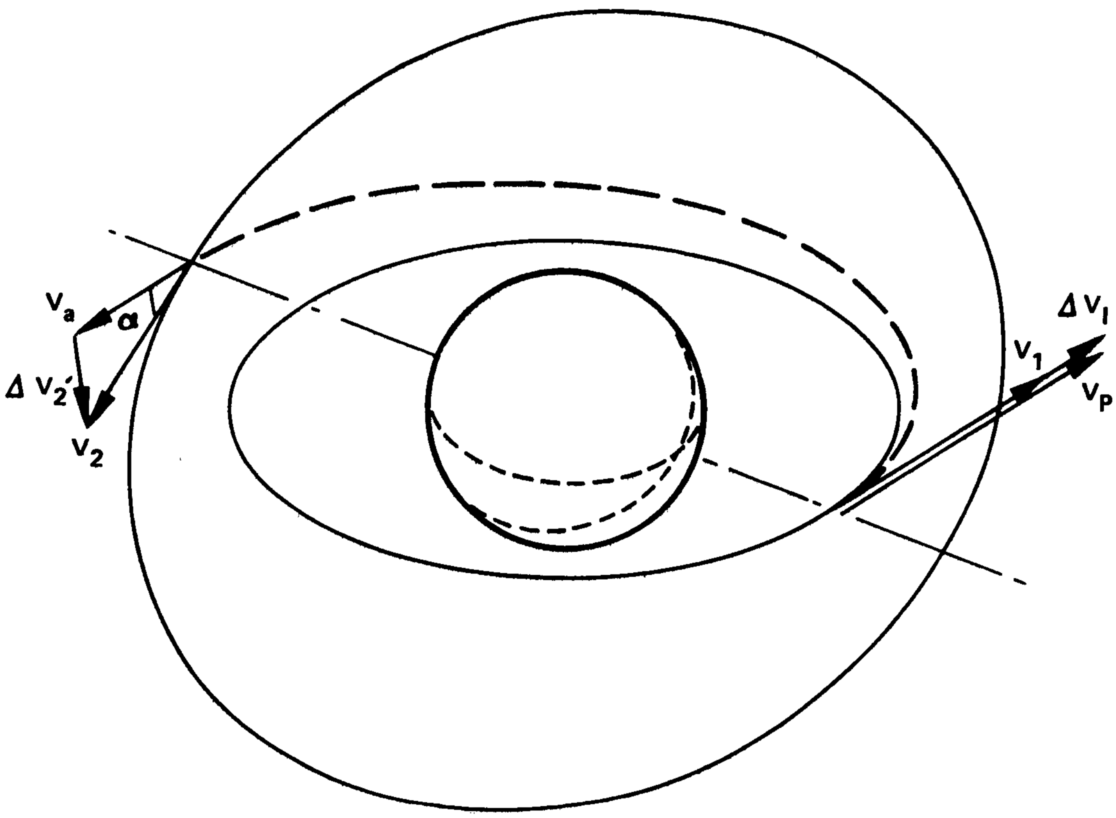

FIGURE 6 - TWO-IMPULSE TRANSFER BETWEEN NONCOPLANAR CIRCULAR ORBITS

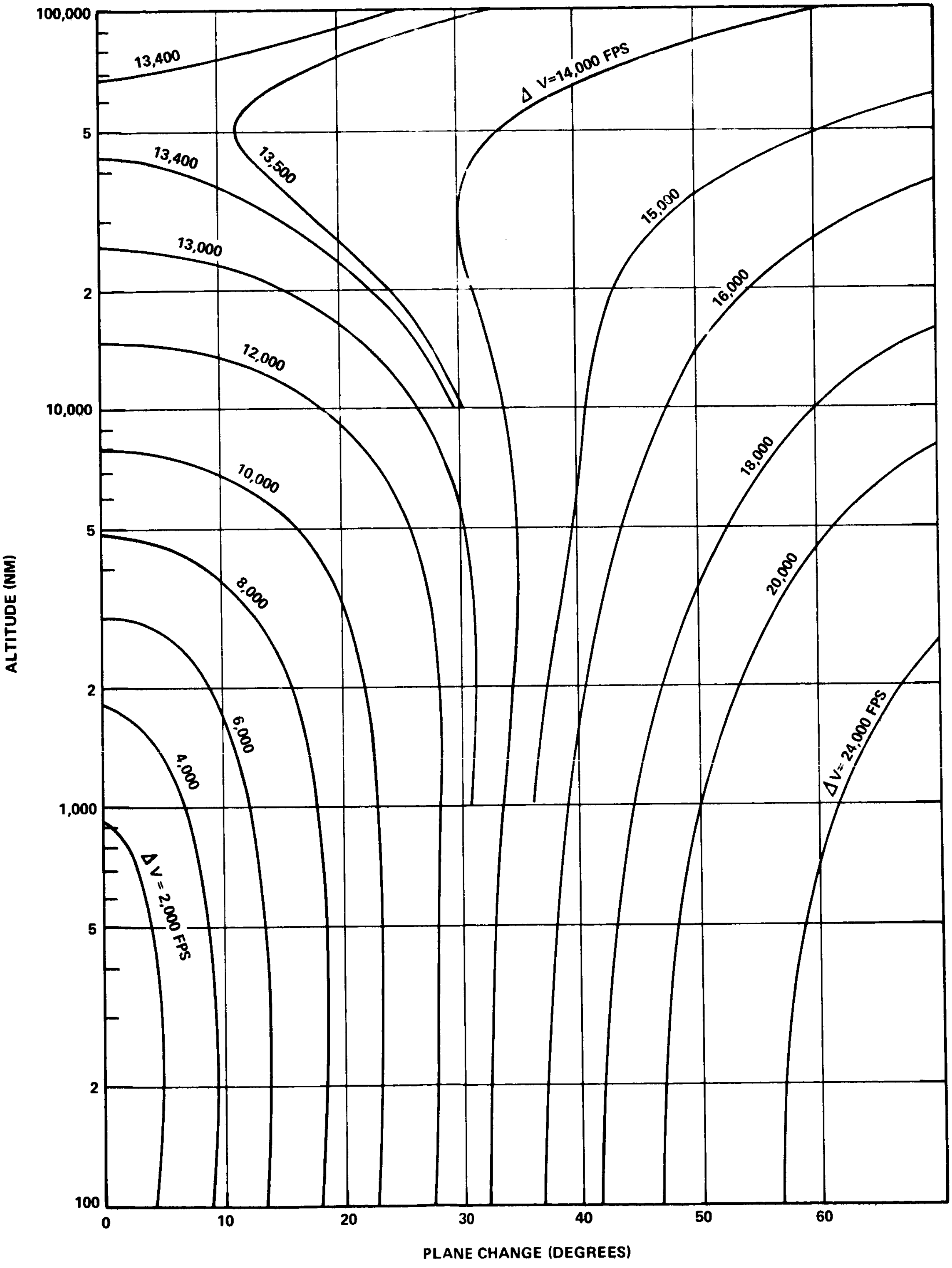

FIGURE 7- VELOCITY REQUIREMENTS FOR NONCOPLANAR TRANSFER FROM A 250 NAUTICAL MILE CIRCULAR EARTH ORBIT TO OTHER CIRCULAR ORBITS

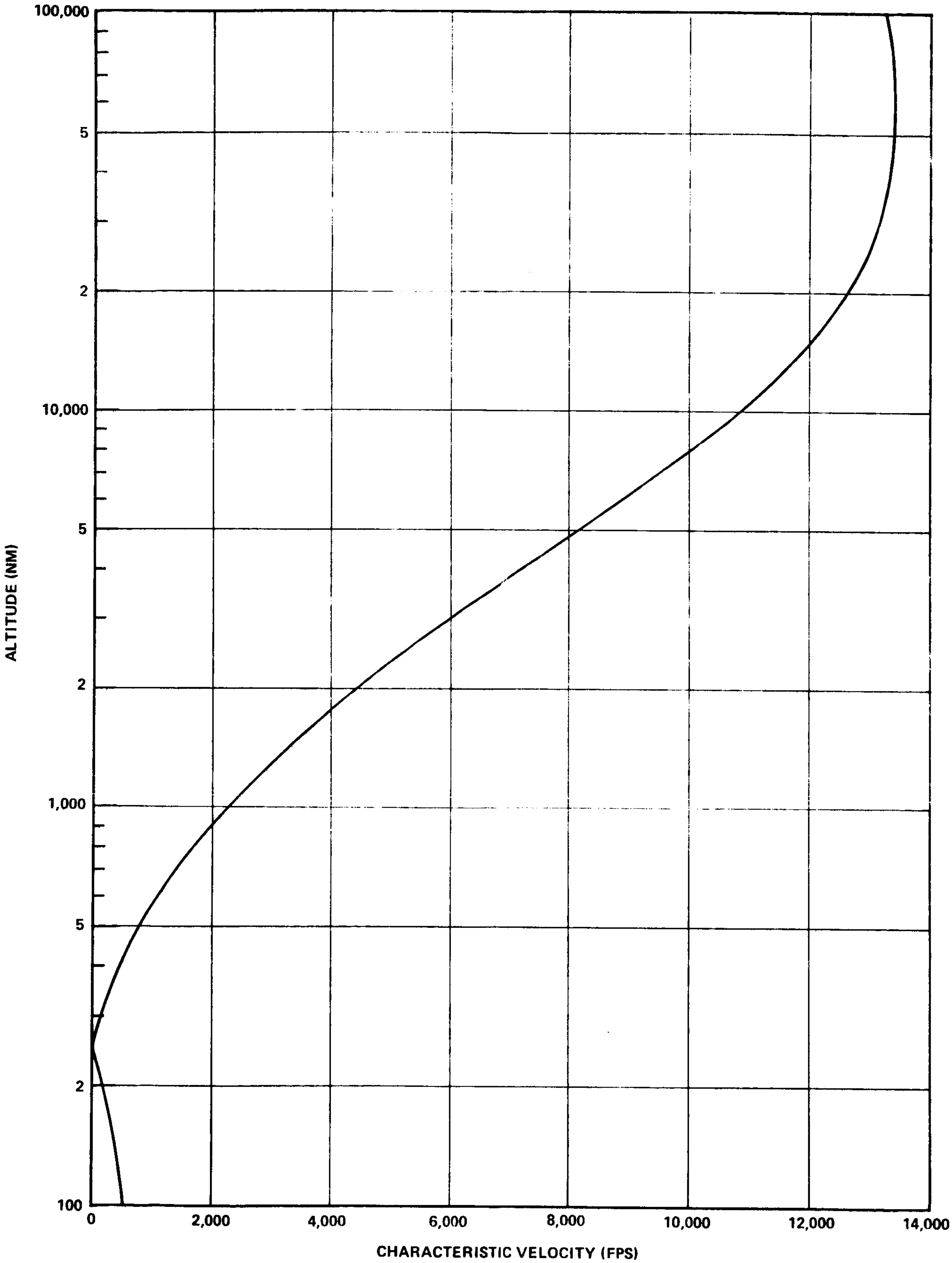

FIGURE 8 - VELOCITY REQUIREMENTS FOR TRANSFER FROM A 250 NAUTICAL MILE CIRCULAR EARTH ORBIT TO OTHER, COPLANAR, CIRCULAR ORBITS

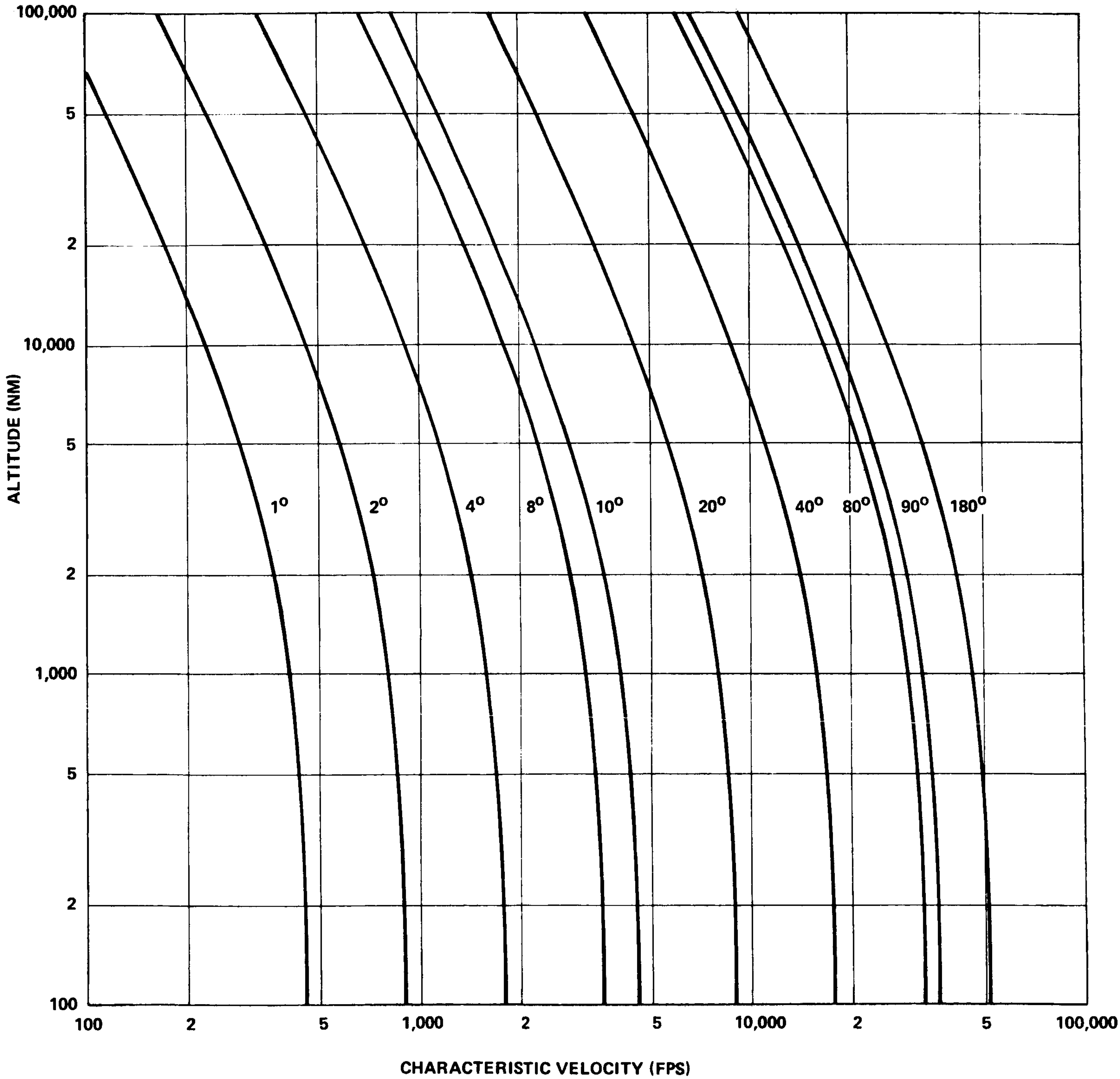

FIGURE 9 - VELOCITY REQUIREMENTS FOR PURE PLANE CHANGE IN CIRCULAR EARTH ORBITS

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